Key

Math 4 Name________________________

1-2 Composition of Functions Date________

- *I can use the definitions and important properties of arithmetic operations, including function composition, in order to combine functions.*

1. Use the function $f(x)=4x^3+7x^2-12x$ to evaluate $f(-2)$. Explain what that tells you about the graph of $f(x)$.

$f(-2)=4(-2)^3+7(-2)^2-12(-2)=20$

The graph goes through the point $(-2, 20)$.

2. Suppose $h(t)=4+90t-16t^2$ represents the height (in feet) of a baseball t seconds after it is hit. Explain what $h(3)=130$ tells us about the situation.

After 3 seconds, the ball is 130 feet in the air.

3. If $f(x)=3x^2-2$ and $g(x)=x+5$, recall that $[f+g](7)=f(7)+g(7)=3(7)^2-2+(7)+5=157$. Find the solutions to the problems below.

a. $[f+g](-2)=$

$=f(-2)+g(-2)$

$=10+3=\boxed{13}$

b. $[f+g](x)=$

$=f(x)+g(x)$

$=3x^2-2+x+5=\boxed{3x^2+x+3}$

c. $[f-g](5)=$

$=f(5)-g(5)$

$=73-10=\boxed{63}$

d. $[f-g](x)=$

$=f(x)-g(x)$

$=3x^2-2-(x+5)=\boxed{3x^2-x-7}$

e. $[f\cdot g](-4)=$

$=f(-4)\cdot g(-4)$

$=46\cdot 1=\boxed{46}$

f. $[f\cdot g](x)=$

$=f(x)\cdot g(x)$

$=(3x^2-2)(x+5)$

$=\boxed{3x^3+15x^2-2x-10}$

g. $[f\div g](7)=$

$=f(7)\div g(7)$

$=\dfrac{145}{12}=\boxed{12.08\overline{3}}$

h. $[f\div g](x)=$

$=\dfrac{f(x)}{g(x)}=\boxed{\dfrac{3x^2-2}{x+5}}$

4. Use the information below to help answer the questions.
   - The annual net income from lottery operation (in millions of dollars) depends on the state's adult population p (in millions) according to the function: $I(p) = 25p - 7.5$.
   - The state's adult population t years from now is predicted by the function: $p(t) = 5(1.02)^t$.

   a. Explain what you would have to do to find the net income from lottery operation in 5 years.

   First find the population (p) in 5 years then plug that into the I(p) function.

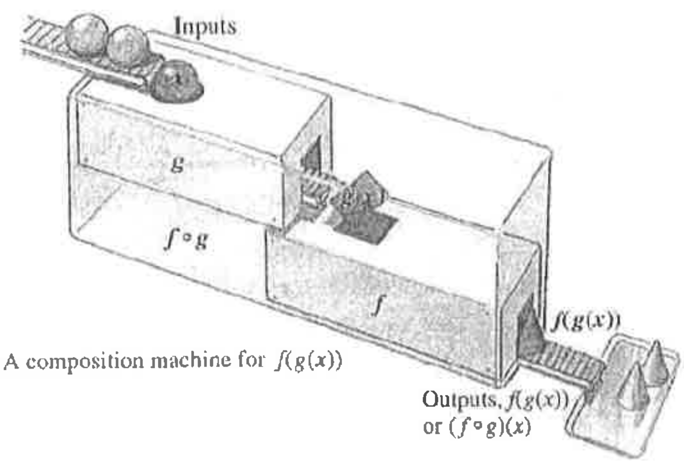
   b. Find the net income from lottery operation in 5 years.

   $p(5) = 5(1.02)^5 \approx 5.52$ million $\quad I(5.52) = 25(5.52) - 7.5 = \boxed{\$130.5 \text{ million}}$

   c. Write a single algebraic rule that can be used to calculate the annual net income from lottery operation t years in the future. In other words, express net income as a function of t.

   $$I(t) = 25(\underbrace{5(1.02)^t}_{P}) - 7.5$$

Your answer to 4c involved combining two functions by an operation called composition. The **composition of functions** is defined as $f \circ g(x) = f(g(x))$ and involves using the output of one function as the input of the other. This is illustrated [redacted] to the right.



A composition machine for $f(g(x))$

Example: If $f(x) = 3.5x$ and $g(x) = 5 - x$, the rule for the composite $f \circ g$ is $f(g(x)) = 3.5(5 - x)$.
To find a specific solution is simple:
$f(g(10)) = 3.5(5 - (10)) = -17.5$

Practice Problems

5. $f(x) = 2x + 5$ and $g(x) = 3x - 1$

   a. Find $f(g(3))$.

   $= f(3(3) - 1) = f(8) = 2(8) + 5 = \boxed{21}$

   b. Find $f(g(x))$.

   $f(3x-1) = 2(3x-1) + 5 = 6x - 2 + 5 = \boxed{6x+3}$

   c. Find $(g \circ f)(x)$.

   $g(2x+5) = g(f(x)) = 3(2x+5) - 1 = 6x + 15 - 1 = \boxed{6x+14}$

6. $f(x) = x+1$ and $g(x) = x^2 + 2x$

a. Find $f(g(-1))$.

$= f((-1)^2+2(-1)) = f(1-2) = f(-1) = -1+1 = \boxed{0}$

b. Find $f(g(x))$.

$f(x^2+2x) = (x^2+2x)+1 = \boxed{x^2+2x+1}$

c. Find $(g \circ f)(x)$.

$g(x+1) = (x+1)^2 + 2(x+1) = x^2+2x+1+2x+2$

$= \boxed{x^2+4x+3}$

7. $f(x) = x^2 + 4x + 3$ and $g(x) = 2x+1$

a. Find $f(g(6))$.

$= f(2(6)+1) = f(13) = (13)^2 + 4(13)+3 = \boxed{224}$

b. Find $f(g(x))$.

$f(2x+1) = (2x+1)^2 + 4(2x+1)+3 = 4x^2+4x+1+8x+4+3$

$= \boxed{4x^2+12x+8}$

c. Find $(g \circ f)(x)$.

$g(x^2+4x+3) = 2(x^2+4x+3)+1 = 2x^2+8x+6+1 = \boxed{2x^2+8x+7}$

8. $f(x) = \frac{1}{x}$ and $g(x) = 2x+3$

a. Find $f(g(-4))$.

$= f(2(-4)+3) = f(-5) = \frac{1}{-5} = \boxed{-\frac{1}{5}}$

b. Find $f(g(x))$.

$f(2x+3) = \boxed{\frac{1}{2x+3}}$

c. Find $(g \circ f)(x)$.

$g\left(\frac{1}{x}\right) = 2\left(\frac{1}{x}\right)+3 = \boxed{\frac{2}{x}+3}$

9. $s(t) = t^2$ and $r(t) = \sqrt{t}$,

a. Find $s(r(9))$.

$= S(\sqrt{9}) = S(\pm 3) = (\pm 3)^2 = \boxed{9}$

b. Find $s(r(t))$.

$S(\sqrt{t}) = (\sqrt{t})^2 = \boxed{t}$

c. Find $(r \circ s)(t)$.

$r(t^2) = \sqrt{t^2} = \boxed{t}$

10. Compare all of the Part b and c answers to Questions 5-9. How is 9 different from the rest?

Parts b + c have the same result.

a. What vocabulary word from last year describes the relationship between $s(t)$ and $r(t)$?

They are inverses of each other.

b. Find another pair of functions that have the same relationship.

Answers vary! $t(x) = x + 1$ + $b(x) = x - 1$

11. Find two functions $h(x)$ and $k(x)$ that compose to give $h(k(x)) = \sqrt{3x-5}$, where $h(x) \neq x$ and $k(x) \neq x$.

$h(x) = \sqrt{x}$

$h(3x-5) =$ ✓

$k(x) = 3x - 5$

12. Find two functions $f(x)$ and $g(x)$ that compose to give $f(g(x)) = \dfrac{3}{x-4}$, where $f(x) \neq x$ and $g(x) \neq x$.

$f(x) = \dfrac{3}{x}$

$f(x-4) =$ ✓

$g(x) = x - 4$